**Carleton University, Computer Science, Fall 2023**

**Comp 1008 Math for Game Programmers**

**Lectures/Instructor:** Tuesday 6-9pm, **W.R. Lalonde**

**Weekly Assignments**: Due Wednesday at Midnight (for assignment provided the week before).

**Course Goal**: Provides the math background for subsequent game programming courses.

**Course calendar description**: Math for building 3D games. Points, vectors, normals. Dot and cross products. Transformations and inverses in left- and right-handed systems. Uses for controlling objects, cameras, and texture manipulation. Bounding boxes, planes, frustums for collision detection and visibility, fast billboarding techniques, point and sphere sweeping. Quaternions.

**Prerequisite(s):** One Grade 12 university preparation mathematics course.

**Course Work**: 1 weekly assignment (70%), in class paper test (10%), and in class exam (20%).

**Brightspace Course materials:** Powerpoint notes; **Pdf NOT USED since slides are animated**.

**Book**: Reference (not required) **Mathematics for 3D Game Programming & Computer Graphics (Third edition is the latest)**, Eric Lengyel, Charles River Media, Inc.

**Handing in**: Assignment files should be handed into Brightspace before midnight on Wednesday (for assignment provided the week before). If the assignments are handed in late, Brightspace will not accept it. Files can be PowerPoint, Word, or Text files (the advantage of PowerPoint is it allows diagrams).

**Discord Link:** [**https://discord.gg/2vwYZXtQ5y**](https://discord.gg/2vwYZXtQ5y) **(icon contains C1-A)**

**Web page:** <http://www.scs.carleton.ca/~lalonde>

**TAs (Marking only): Francis Chen, Robertson Riley**

Course Outline

The rendering pipeline

 Generic overview

 Left versus right-handed coordinate systems

 Left-to-right versus right-to-left evaluation math

 Coordinates spaces (object, world, camera, and perspective)

Tuples

 The distinction between points, vectors, normals

 Two definitions of vector dot product

 Two definitions of vector cross product

 Intuitions behind dot products and cross products

 Many operations and related theorems on tuples, points, vectors, dot products and cross products.

Matrices

 Matrix multiply, matrix transposes, and vector-matrix operations

 Matrix inverses and how to compute them

 Useful theorems involving transposes and inverses

 Matrix forms of dot and cross products

Transformations

 Translating, rotating, scaling transformations and their inverses

 Projection transformations

 Properties of rotations

 The general rotation transformation

 Fast inverses

Transformations for gaming

 Placement matrices versus Delta matrices in the context of rotating (R), scaling (S), and translation (T).

 Controlling placement by using the natural order SRT

 Object placement in worlds

 Texture placement in textured objects

 Camera placement in worlds

 Controlling and animating changes via delta matrices

 Changes relative to an Object (pre-transformations)

 Changes relative to a World (post-transformations)

 Changes relative to a Parent

 Controlling changes when dealing with inverses

 Placement and Delta Changes in right to left-handed systems

 Articulated figures, poses, and skinning

 Object and camera “look at” functions

Visibility determination

 Bounding boxes, planes, frustums, and visibility trees

 Octrees, quad trees, bounding box trees, and bsp trees.

 Distance to planes, plane transformations

 Building frustums, frustum transformations

 Determining if points, bounding boxes, and spheres are inside frustums

 Sprites and how to draw them without needing to rotate them

 Portal visibility

Collision detection and collision reacting

 Movement boxes and collision detection trees (similar to visibility trees)

 Basic algorithms for projections of points on a line, on a plane

 Basic algorithm for intersection of a line with a plane, with a sphere

 The concept of object sweeping.

 Point sweeps colliding with planes, spheres, bounding boxes, polygons, and polygon soup.

 Sphere sweeps colliding with planes, spheres, bounding boxes, polygons, and polygon soup.

 More general sweeps done efficiently

Odd and ends

 Quaternions if time admits

Course Takeaway

 Deep intuitive and mathematical understanding of vectors and transformations. Less mathematical but

 still intuitive understanding of visibility and collision detection.

**Artificial Intelligence**

With the advent of large language models (LLMs), it’s becoming more and more possible to use chatbots such as ChatGPT, Google Bard, Bing Chart, research assistants such as Elicit, and image generators such as Stable Diffusion, Dall-E to aid with course work. However, there are problems with those tools:

* They sometimes hallucinate; i.e. lie.
* They can’t explain why something is true.
* They can’t attribute knowledge (they don’t know where they found this information).
* They can’t learn.
* They don’t know if they don’t know something; i.e., always give an answer whether right or wrong.
* They don’t understand non-linguistic knowledge; e.g., liquids filling containers until full, batteries discharging until they don’t work, 3D relationships between people and objects (e.g., right, left, above, below).
* They have no sense of ethics, danger, or reasonableness (e.g., a 1000 year old person).

This course was designed to be completed by individuals working alone. For this reason, tests and exams are given in class without in-class access to external tools or notes.

However, in the interests of societal advancement, we are interested in whether or not such tools should ultimately become part of the curriculum. Consequently, we will allow the use of such tools to check the correctness of your solutions provided they are fully attributed. For example, “this assignment was checked by XXXX and found 3 problems; problem 3 was originally solved as YYY which I revised to ZZZ, problem 7 …, problem 12 …” TAs marking the assignments will make that information available to the lecturer who may divulge the results to the class if it proves interesting.

**University Policies**

**Student Academic Integrity Policy**

Every student should be familiar with the Carleton University student academic integrity policy. A student found in violation of academic integrity standards may be awarded penalties which range from a reprimand to receiving a grade of F in the course or even being expelled from the program or University. Some examples of offences are: plagiarism and unauthorized co-operation or collaboration. Information on this policy may be found in the Undergraduate Calendar.

**Plagiarism**

As defined by Senate, "plagiarism is presenting, whether intentional or not, the ideas, expression of ideas or work of others as one's own". Such reported offences will be reviewed by the office of the Dean of Science.

Unauthorized Co-operation or Collaboration

Senate policy states that "to ensure fairness and equity in assessment of term work, students shall not co-operate or collaborate in the completion of an academic assignment, in whole or in part, when the instructor has indicated that the assignment is to be completed on an individual basis". Please refer to the course outline statement or the instructor concerning this issue.

**Academic Accommodations for Students with Disabilities**

The Paul Menton Centre for Students with Disabilities (PMC) provides services to students with Learning Disabilities (LD), psychiatric/mental health disabilities, Attention Deficit Hyperactivity Disorder (ADHD), Autism Spectrum Disorders (ASD), chronic medical conditions, and impairments in mobility, hearing, and vision. If you have a disability requiring academic accommodations in this course, please contact PMC at 613-520-6608 or pmc@carleton.ca for a formal evaluation. If you are already registered with the PMC, contact your PMC coordinator to send me your Letter of Accommodation at the beginning of the term, and no later than two weeks before the first in-class scheduled test or exam requiring accommodation (if applicable). After requesting accommodation from PMC, meet with me to ensure accommodation arrangements are made. Please consult the PMC website for the deadline to request accommodations for the formally-scheduled exam (if applicable) at http://www2.carleton.ca/pmc/new-and-current-students/dates-and-deadlines

**Religious Obligation**

Write to me with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details visit the Equity Services website: http://www2.carleton.ca/equity/

**Pregnancy Obligation**

Write to me with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details visit the Equity Services website: http://www2.carleton.ca/equity/

**Medical Certificate**

The following is a link to the official medical certificate accepted by Carleton University for the deferral of final examinations or assignments in undergraduate courses. To access the form, please go to http://www.carleton.ca/registrar/forms