Minimizing the Maximum Sensor Movement for Barrier Coverage of a Line Segment with Disjoint Sub-Intervals
Abstract

Mobile sensors along a barrier allows for the detection of an intruder trying to pass through the barrier. Initial sensor placement can not always be done in an optimal way and the sensor must then be moved to provide complete and optimal coverage of the barrier. Moving the sensors can be costly to their battery power and so the movement to the optimal positioning must be done in such a way that each sensor moves a minimal distance. The project considers when the barrier contains segments that are unnecessary for the sensors to cover and attempts to devise algorithms to move the sensors to an optimal positioning.
Acknowledgements

A thank you to Professor Kranakis for introducing me to this problem and the help he had provided during the project.

All materials used in this project not owned by myself are cited inline and a list of references can be found at the end of the report in the References section.
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1 Introduction

Infrastructure plays a critical role in the operation of societies and enterprises. Roads, buildings, border systems, telecommunication systems are all examples of infrastructure that necessitates the need for a security system to be in place. The security system must be able to withstand and recover from potential threats that could affect the critical infrastructure within a given bounded region [Kranakis and Krizanc, 2015].

There are four topics that have been studied quite extensively in recent years; patrolling, sensor coverage and interference, evacuation and, domain protection and blocking [Kranakis and Krizanc, 2015]. Patrolling is about surveilling an area with mobile agents who move around the area to observe and protect that area. One problem that is studied in patrolling is finding the optimal movements of the monitoring agents to ensure that no critical point in the area is left unmonitored longer than the allowed idle time and to ensure that if a subset of the total agents are faulty or untrustworthy that the critical points are still monitored by trustworthy agents. Sensor coverage and interference is about monitoring an area with mobile sensors that can retrieve data from and observe changes in the monitored area. Two problems that are studied in sensor coverage are the area coverage, ensuring full coverage of the vital area, and perimeter or barrier coverage, ensuring coverage of the perimeter of the area. Both problems involve studying how to minimize the displacement of sensors from a position without complete coverage to a position that gives complete coverage. Evacuation involves agents trying to evacuate a given area without prior knowledge of where the exit is located. One problem studied is how to minimize the time it takes for all the agents to find the exit. Domain protection and blocking is about the detection of intruders passing through a given area or perimeter that has sensor placed at regularly spaced intervals. The problem that is studied is that over time these sensors may fail and it is desirable to know if detection of intrusions will still happen using the remaining functional sensors.

This project is focused on a sub-problem of the barrier coverage problem which is concerned with ensuring that the perimeter of an area is monitored by sensors. If complete coverage of a barrier is achieved it can detect any entity entering or exiting the region. This differs from complete area coverage which can detect an intruder within the region [Czyzowicz et al., 2010]. Barrier coverage is often more desirable than area coverage as fewer sensors are required to give a complete coverage of the perimeter than it is to give a complete area which means a lower overall cost [Chen et al., 2013].

The sub-problem is the introduction of special segments, referred to as neutral segments [Collins et al., 2013], on the barrier which are either unnecessary to monitor or are impossible to monitor due to external forces, such as if the sensors monitoring interferes with other infrastructures. The segments that are not neutral segments need to have full coverage for the entire barrier to be considered completely covered. We refer to these segments that need to be covered as vital segments [Collins et al., 2013]. The setting of the problem includes a bounded region and a set of sensors that monitor the perimeter of the region. Each sensor has a position that can be within or outside of the region and has a pre-define sensing range allowing it to
monitor all point within that range. The initial positions of the sensors are arbitrary and the goal is move the sensors onto positions on the boundary such that complete coverage, or at least maximal coverage, is gained.

This project looks at the optimization problem of minimizing the movement of each sensor from an initial position to a position along the barrier achieving maximal coverage while working around the neutral segments of the perimeter. This problem arises when the initial placement of the sensors is not optimal [Czyzowicz et al., 2010], potentially due to environmental factors such as terrain or danger to the placer. It can also arise if a sensor fails and a readjustment is required to regain maximal coverage. When either of these situations occur, it is necessary to find the minimal movement to regain an optimal placement to preserve sensor resources.

This project restricts the problem to a one-dimensional version where a finite line segment is considered as the barrier. We consider the movement of the sensors to be along the line and that the initial positions of the sensors are also on the line.

1.1 Notation and Preliminaries

We define a barrier as a cyclic closed line interval with predefined points as $I = [0, L]$ where $L > 0$ on the real number line representing the perimeter. The interval $I$ consists of a set of vital segments, denoted as $V$, and a set of neutral segments, denoted as $N$, such that $I = \bigcup_{i=1}^{|V|} V_i \cup \bigcup_{j=1}^{|N|} N_j$. There is no overlap between vital and neutral segments other than the end points of each segment. The interval $I$ will always begin with a vital segment as if the interval began with a neutral segment we could reduce the scenario to one where that neutral segment is at the end of the interval $I$ due to the cyclic property of the perimeter. We assume in this project that vital segments cannot be adjacent to each other as they could be reduced to a vital segment that is the union of the two segments. We will also denote the sum of the vital segment lengths as $L^* = \sum_{i=1}^{|V|} |V_i|$. This value is the total length that must be monitored by the sensors for complete coverage to be achieved.

When considering a neutral segment $N_i$, where $1 \leq i \leq |N|$, we refer to it as unnecessary when the segment does not require to be monitored and as impossible when the segment cannot be monitored. Neutral segments are assumed to not be adjacent as two impossible segments can be reduce to a union of the two and two unnecessary segments could have the same done. We assume that the case where an impossible and an unnecessary segment are adjacent is not possible, though this could be studied in the future.

For an instance of the problem we have $n$ sensors in the set of sensors $S$. We define an individual movable sensor $s_i$ to have a position $x_i$ that has a sensing radius of $r_i$. An individual sensor can monitor the segment represented by the interval $[x_i - r_i, x_i + r_i]$. We denote the range interval of a sensor $s_i$ as $\text{range}(s_i) = [x_i - r_i, x_i + r_i]$ and the sum of all sensor ranges as
\[ R = \sum_{i=1}^{[S]} r_i, \] which is equal to \(2rn\) when sensor ranges are equal. We consider the order of the sensors in the set by their initial position so that sensors with a lower initial position come before those with a higher initial position, i.e. \(x_1 \leq x_2 \leq \cdots \leq x_n\). The initial placement and range of a sensor may potentially not lie on the barrier.

There will cases where we need to move a sensor. When we are referring to the movement of a sensor we will use the notation \(\text{move}(s, x)\) where \(s\) is the sensor and \(x\) is the new position of the sensor such that the range of the sensor covers \([x - r, x + r]\) where \(r\) is the radius of the sensor \(s\).

There will be a case where we need to move a sensor minimally to cover a smaller segment. When that happens, we will use the notation \(\text{cover}(s, \text{seg})\) where \(s\) is the sensor we are moving and \(\text{seg}\) is the segment that the sensor must cover in its sensing range. This will only be done when \(\text{seg}\) is a segment of length less than or equal to the range of the sensor \(s\). This leads to the development of the following algorithm that makes a minimal move of the sensor to cover the segment \(\text{seg}\). The \(\text{lower}(s)\) and \(\text{upper}(s)\) functions return the lower and upper bound of a given segment or interval respectively.

**ALGORITHM 1.1.1:**

\[
\text{cover}(s, \text{seg}):
\]

\[
\text{if} \ \text{lower}(s) > \text{lower}(\text{seg}) \ \text{then} \\
\quad \text{move}(s, \text{lower}(\text{seg}) + r) \\
\text{else if} \ \text{upper}(s) < \text{upper}(\text{seg}) \ \text{then} \\
\quad \text{move}(s, \text{upper}(\text{seg}) - r) \\
\text{else} \\
\quad \text{do nothing as } s \text{ already covers } \text{seg}
\]

This algorithm works as we only wish to move a sensor with a larger range segment to cover a smaller segment. We shift the sensor so the lower bound of the range is the same as the lower bound of the smaller segment or shift the upper bound of the range to have the same range as the upper bound of the smaller segment.

For a vital segment, we will want to know how many sensor can fit in the segment. For this we will define a function, \(\Lambda(V_i)\), that returns a tuple that contains the information desired. In the case where sensor ranges are the same the function will return a 2-tuple that contains the number of sensors that can fit in the segment without overlap and the remaining length that would not be covered by the given number of sensors. Here we make use of \textbf{quo} and \textbf{mod} and these are used else where as well. The usage of \textbf{quo} denotes integer division, i.e. \(x \ \textbf{quo} \ y = \left\lfloor \frac{x}{y} \right\rfloor\). The usage of \textbf{mod} denotes the modulo operator.
\[ \Lambda(V_i): \]
\[
\begin{align*}
\sigma & \leftarrow |V_i| \text{ quo } 2r \\
\rho & \leftarrow |V_i| \text{ mod } 2r \\
\text{return } (\sigma, \rho)
\end{align*}
\]

To short hand the values of the 2-tuple the notation \( \Lambda(V_i).\sigma \) and \( \Lambda(V_i).\rho \) will be used to get the number of sensor that can fit without overlap and the remaining length respectively.

Now a problem arises if \( n > \sum_{i=1}^{|V|} \Lambda(V_i).\sigma \) which means there are more sensors then there are positions available vital segments that can be moved to without a loss of range. To handle the distribution of the remaining sensors we must consider each \( V_i \) and determine which has the greatest remaining length that needs to be monitored. We do this by use of the \textit{adjusted}\( \Lambda \) algorithm which gives back a set of modified \( \Lambda \) 2-tuples where the \( \sigma' \) greatest tuples have their \( \sigma \) value incremented by one to denote that giving the associated vital segment will maximize the barrier coverage. The \textit{greatest}(\( \sigma', V_i \)) function in the algorithm determines if the given \( V_i \) is one of the vital segments that has one of the \( \sigma' \) greatest remaining length to cover by considering the \( \Lambda(V_i).\rho \) value.

\[ \text{adjusted}\( \Lambda(V, \sigma') \):} \]
\[
\begin{align*}
\text{for } i \text{ from 1 to } |V| \text{ do} & \\
\text{if greatest}(\sigma', V_i) & \text{ then} \\
\Lambda'(V_i).\sigma & \leftarrow \Lambda(V_i).\sigma + 1 \\
\Lambda'(V_i).\rho & \leftarrow 2r - \Lambda(V_i).\rho \\
\text{else} & \text{ end if} \\
\Lambda'(V_i) & \leftarrow \Lambda(V_i) \\
\text{end if} \\
\text{return } \Lambda'
\end{align*}
\]

This algorithm runs in linear time since it is only dependant on the size of the set of vital segments.

When a figure appears in this document it will follow the following example:
1.2 Problems

We now define a few variants of the barrier coverage problem with neutral segments.

The first variant to consider is when $R < L^*$. For this variant, complete coverage is not possible as there are not enough sensors to provide complete coverage to all the vital segments. We are then interested in studying the optimization problem of finding the maximal coverage while minimizing the movement of the sensors. In the variation of the problem where there are not neutral segments there are two variations of the solution [Czyzowicz, 2010]. The first being the contiguous case where the largest sub interval of $I$ possible is found and the second being the non-contiguous case where the largest possible coverage of potentially disjoint sensors is found. In the case were neutral segments are included, a contiguous solution is often not possible due to the neutral segments and will not be studied in this project. The non-contiguous case will be studied when we discuss this variant.

The second variant to consider is when $R = L^*$. For this variant, complete coverage is possible but only when certain conditions hold. We are interested in finding these conditions and then solving the optimization problem of minimizing the movement of sensors when complete coverage is possible and when complete coverage is not possible.

The third variant to consider is when $R > L^*$. For this variant, like the $R = L^*$ variant, complete coverage is possible only when certain conditions hold. We are interested in finding
these conditions and then solving the optimization problem of minimizing the movement of sensors when complete coverage is possible and when complete coverage is not possible.

1.3 Related Work

There are several papers that study the barrier coverage problem. In [Czyzowicz et al., 2010] the problem is introduced for a one-dimensional interval and gives $O(n)$ time optimal algorithms for the $R < L$ contiguous and non-contiguous variations and the $R = L$ variation when the sensor ranges are equal. It also gives a $O(n^2)$ time optimal algorithm for the $R > L$ variation as well as two approximation algorithms that have a lower time complexity when the sensor ranges are equal. In [Chen et al., 2013] they present a $O(n^2 \log n)$ time algorithm for the for $R > L$ when sensors have arbitrary range and an improved $O(n \log n)$ time algorithm for when sensors ranges are equal. In [Kranakis et al., 2013] they look at the maximum displacement of sensor to achieve coverage in both one-dimensional and two-dimensional space. In [He and Zhang, 2016] they look at the barrier coverage problem in two-dimensional space and how the barrier can be completely covered with the sensor being close to the interval instead of on the interval. In [Fan et al., 2014] they consider the problem with sensors that have adjustable ranges determined by a power level that the sensor chooses, giving results for when the power levels are chosen form a finite set of powers and when the powers are chosen from a range of powers. In [Eftekhari et al., 2013] they study the barrier problem in a distributed setting where each sensor is autonomous and finds the position only knowing their immediate surroundings.

In the paper [Collins et al. 2013] they present optimal patrolling using mobile agents on a fragmented boundary. The concept of neutral and vital segments is from paper.

This project differs from the mentioned as it applies the fragmented barrier concept and applies it to the barrier coverage problem.

1.4 Outline of the Report

In this report, we explore each of the sub-problems presented in Section 1.2. In Section 2 we explore when the sum of the sensor ranges is less than the sum of the vital segments lengths, meaning that the sensors cannot provide full coverage. In Section 3 we explore when the sum of the sensor ranges is equal to the sum of the vital segment lengths. In Section 4 we explore when the sum of the sensor ranges is greater than the sum of the vital segment lengths.

For each sub-problem, the continuous interval variation of the problem will be introduced then expanded to the disjoint interval variation and a devised algorithm to move the sensors will be given and its efficiency will be examined.
2 Optimization Problem for $R < L^*$

In this variation of the problem the sum of the sensor ranges is less than the total area that the vital segments cover. There are two case that arise when we look at this problem. The first is when the sensors can be moved into positions where there is not loss of sensor range, i.e. no overlap between sensors or overlap with a neutral segment that is unnecessary to monitor. The other is when there will be a loss due to an overlap.

The following figure shows how the positioning of the sensors can result in a loss of range. The range is wasted on the first line as the third sensor is placed to monitor the largest remaining area but it ends up overlapping with an unnecessary segment. The second line shows when there is no need for overlapping sensors and a maximal coverage is gained but there is the unmonitored part of the segment near the first sensor.

![Figure 2: Example of Range Loss in $R < L^*$](image)

Theorem 2.1 – There is no range loss if and only if $\sum_{i=1}^{\lvert V \rvert} \Lambda(V_i) \cdot \sigma \geq n$

Proof:

Consider that you have $n$ sensors and let $\alpha = \sum_{i=1}^{\lvert V \rvert} \Lambda(V_i) \cdot \sigma$ and $\beta = \sum_{i=1}^{\lvert V \rvert} \Lambda(V_i) \cdot \rho$. In the case where $\alpha \geq n$ you know that the sensors can be moved to some position on the barrier such that no loss will occur since $\alpha$ represents the number of places where a sensor will fit perfectly without any loss.

If $\alpha - n > 0$ then we know that we did not have enough sensors to fill all the vital segments and that no sensor was moved in such a way that range overlapped with an unnecessary segment or another sensor.

If $\alpha - n = 0$ then we must consider $\beta$. If $\beta = 0$ then $R = L^*$ and no loss would have occurred because all the positons where a sensor could fit without loss have been filled and there are no vital segments with a length that cannot be evenly by sensors. If $\beta > 0$ then we know we did not have enough sensors for complete coverage but no sensor range would have overlapped with another sensor or an unnecessary segment since only positons where a sensor could without loss were filled.
In the case $a < n$ we know that all positons where a sensor could fit without loss have been filled. This means that $n - \alpha$ sensors must still be moved to positions in some vital segment. Since the only space in each vital segment $V_i$ is $\Lambda(V_i) \rho$ we know a loss will occur either by overlap with another sensor or an unnecessary segment since $\Lambda(V_i) \rho < 2r$.

This algorithm simply reduces each vital segment to its own $MinMax$ problem. The $MinMax$ algorithms are presented in [Czyzowicz et al., 2010] and can be used to solve the barrier coverage problem without neutral segments.

The presented algorithm first determines if there are any sensors that cannot be placed without overlapping another sensor or unnecessary segment. This value is denoted as $\sigma'$. If there are sensors that cannot be moved to a position without overlap, i.e. $\sigma' > 0$, then the values of $\Lambda(V_i)$ must be adjusted to account for these sensors. Once this is done the problem is then reduced and the appropriate $MinMax$ algorithm is used given the interval to perform the $MinMax$ on with the set of sensors that must be moves and their initial positons.

ALGORITHM 2.1:

\[
\begin{align*}
\sigma' &\leftarrow n - \sum_{i=1}^{\left|V\right|} \Lambda(V_i).\sigma \\
\text{if } \sigma' &> 0 \text{ then} \quad \Lambda' \leftarrow adjusted\Lambda(V, \sigma') \\
\text{else} \quad \Lambda' &\leftarrow \Lambda \\
\text{end if} \\
\sigma &\leftarrow 1 \\
\text{for } i \text{ from 1 to } \left|V\right| \text{ do} \\
&\quad S^* \leftarrow S_{\sigma}, ..., S_{\sigma+\Lambda'(V_i)\sigma} \\
&\quad X^* \leftarrow x_{\sigma}, ..., x_{\sigma+\Lambda'(V_i)\sigma} \\
&\quad MinMax(V_i, S^*, X^*) \\
\text{done}
\end{align*}
\]
3 Optimization Problem for $R = L^*$

In the variation of the problem where there are no neutral segments a solution can easily be obtained in linear time by moving each sensor $S_i$ to the position $(2i - 1)r$ [Czyzowicz, 2010]. In this variation of the problem, the problem can be reduced to a simple positioning like that if complete coverage is possible. We will present an algorithm to achieve complete coverage if possible and an algorithm to achieve maximal coverage if complete coverage is not possible. The following theorem presents when complete coverage is possible:

Theorem 3.1 – full coverage can be achieved if and only if all vital segment lengths are divisible by $2r$.

Proof:

Let a vital segment have a length that is not divisible by $2r$. Let us refer to this segment as $V_i$ and denote the length of $V_i$ as $|V_i| = 2rk + c$ where $0 \leq k < n$ and $0 < c < 2r$. Thus, we can use $k$ sensor to fill $2rk$ of the segment and are left with $c$ of the segment unmonitored. Let us refer to the neutral segment that between $V_i$ and $V_{i+1}$ as $N_i$. Now we have two cases, the first is where $N_i$ is an unnecessary segment to monitor, the second is where $N_i$ is an impossible segment to monitor. We will denote any sensing range lost, any range of a sensor covering an segment that does not need to be monitored either because it is already monitored or because it is an unnecessary segment to monitor, as $\lambda$.

![FIGURE 3: A Placement when $N_i$ is Unnecessary](image3)

In the case of $N_i$ being an unnecessary segment to monitor, if the next sensor is place to cover the remaining $c$ of the segment then part of the sensing range of that sensor will overlap with $N_i$. If $|N_i| > 2r - c$, in other words the range does not cover any part of the next vital segment on the interval, then $\lambda = 2r - c$, otherwise $\lambda = |N_i|$ if the sensor range covers part of the next vital segment.

![FIGURE 4: A Placement when $N_i$ is Impossible](image4)
In the case of $N_i$ being an impossible segment to cover, the sensor must be placed on either side of the neutral segment. If it is placed on the side so it covers the remaining $c$ then $\lambda = 2r - c$ since that is the amount that overlaps with another sensor range, otherwise $\lambda = c$ since the $c$ remaining to cover in the vital segment was not covered.

This gives us the total coverage as $2rn - \lambda$ and since $\lambda > 0$, then $2rn - \lambda < 2rn$ holds true, which means that the total coverage of $L^* = 2rn$ is not met.

\[\square\]

### 3.1 When Complete Coverage Is Possible

As we saw in Theorem 3.1 complete coverage is only possible when the length of each vital segment is divisible by $2r$. This means that each vital segment $V_i$ can fit $\left\lfloor \frac{|V_i|}{2r} \right\rfloor$ sensors which means that no vital segment has any unmonitored segments. Thus, we can essentially reduce the problem to the algorithm given in [Czyzowicz, 2010]. The difference is that we must keep track of the displacement that is caused by the neutral segments. We can consider the final position of the sensor as follows. Consider the interval of $V_i$ where $1 \leq i \leq |V|$ to be $[\text{lower}V_i, \text{upper}V_i]$ and the set of sensors that have the final position within $V_i$ as $S_j^*$ where $1 \leq j \leq \left\lfloor \frac{|V_i|}{2r} \right\rfloor$; $S_j^*$ is moved to position $\text{lower}V + (2j - 1)r$. This algorithm essentially the placement in each $V_i$ to the algorithm given in [Czyzowicz, 2010].

For this algorithm, we must keep track of the current vital segment we are considering. We can do this by keeping track of the index of the segment within $V$ and we will denote that index with the symbol $v$. We will also need to keep track of our current position along the interval $I$, i.e. the position that we place the lower bound of the next sensor at, let this be denoted as $\delta$. The initial value of $v$ will be 1 and the initial value of $\delta$ will be 0. Let the value $\sigma$ denote the index of the current sensor.
ALGORITHM 3.1.1:

\[ v \leftarrow 1 \]
\[ \delta \leftarrow 0 \]
for \( \sigma \) from 1 to \( n \) do
\begin{align*}
\text{move } S_\sigma \text{ to } & \delta + r \\
\delta & \leftarrow \delta + 2r \\
\text{if } & \delta \geq \text{upper}(V_v) \text{ and } v \leq |N| \text{ then} \\
\delta & \leftarrow \text{upper}(N_v) \\
v & \leftarrow v + 1
\end{align*}
end if

\[ \text{done} \]

It is easy to see that the algorithm runs in \( O(n) \) time as it goes through and moves each sensor.

FIGURE 5: Example of Complete Coverage Movement \( R=L^* \)

3.2 When Complete Coverage is Not Possible

When complete coverage is not possible the goal is to maximize the coverage that is possible while minimizing the maximum sensor movement. Since \( R = L^* \) we know that there will be a loss of sensing range so we try and fill the largest remaining areas of the vital segments with the remaining sensors after the \( \sum_{i=1}^{[V]} \Lambda(V_i) \). \( \sigma \) other sensors have been assigned to a vital segment.

The first step is to determine the number of sensors that are remaining after we find the number of sensors that fit in each vital segment without overlap. This value is denoted by \( \sigma' \) and is determined by subtracting all \( \Lambda(V_i) \). \( \sigma \) from \( n \) the number of sensors. From this we know that \( \sigma' \) sensors must be assigned to segments and to maximize the coverage they must be the...
segments with the greatest remaining length to cover. We will define the modified \( \Lambda(V_i) \) tuples as \( \Lambda'(V_i) \).

The algorithm works by finding \( \sigma' \), which is the number of sensors that will be used to fill the gaps in the vital segments. Then the adjusted \( \Lambda(V_i) \) values are determined to account for the sensors with overlap and the problem is then reduced and the appropriate MinMax algorithm is used given the interval to perform the MinMax on with the set of sensors that must be moves and their initial positons.

**ALGORITHM 3.2.1:**

\[
\begin{align*}
\sigma' & \leftarrow n - \sum_{i=1}^{|V|} \Lambda(V_i) \cdot \sigma \\
\Lambda' & \leftarrow \text{adjusted} \Lambda(V_i, \sigma') \\
\sigma^* & \leftarrow 0 \\
\text{for } i \text{ from } 1 \text{ to } |V| \text{ do} & \\
& \quad S^* \leftarrow S_{\sigma}, \ldots, S_{\sigma + \Lambda'(V_i) \cdot \sigma} \\
& \quad X^* \leftarrow x_{\sigma}, \ldots, x_{\sigma + \Lambda'(V_i) \cdot \sigma} \\
& \quad \text{MinMax}(V_i, S^*, X^*) \\
& \quad \sigma^* \leftarrow \sigma^* + \Lambda'(V_i) \cdot \sigma \\
\text{done}
\end{align*}
\]
4 Optimization Problem for $R > L^*$

This variation of the problem would have two cases like that of the $R = L^*$ variation. The one case being where complete coverage is possible and the other being where complete coverage is not possible. Complete coverage would be possible where there are enough sensors to fill in the gaps in the vital segments. This means that for each $V_i$ where $\Lambda(V_i) \cdot \rho > 0$ there is an additional sensor for that vital segment in addition to the $\sum_{i=1}^{\left| V \right|} \Lambda(V_i) \cdot \sigma$ sensors that are required for full coverage.
5 Summary of Results

In this report, we have presented algorithms to perform the barrier coverage problem on barriers that contain segments that are not required to be monitored by the mobile sensors. We introduced algorithms for the cases where the total sensor range is less than the area needed to be monitored and when the total sensor range is equal to the area needed to be monitored. Ideas are introduced about when the total sensor range is greater but is not explored in detail.

The only algorithm that is optimal that is given is that for the case where the total sensor range is equal to the require area to be cover when each vital segment is divisible by the range of the sensors. The other algorithms present a view into what each problem requires and potentially will lead to an optimal algorithm.
6  Further Work

The solution given for the case where $R < L^*$ is not optimal in all cases. This is because it fills the vital segments from one end to the other which means that depending on the initial placement of the sensors they would pass the vital segment in which they would have an optimal position.

Due to time constraint, the variation of the problem where $R > L^*$ was not fully studied. This variation would be a very interesting to study further and devise an algorithm for.

This project focused on the problem with respect to the sensors having equal ranges. Further study should be done into this problem when the sensors have arbitrary ranges as this has been done for other variations of the problem.

A few of the presented algorithms make use of the $MinMax$ algorithms found in [Czyzowicz et al., 2010]. These algorithms could potentially be better integrated together providing better running time algorithms. The approach of using this algorithm may also not be the best approach to this problem and another avenue could potentially provide with more effective or efficient algorithms.
7 References


